Polynomial Kernels for Traveling Salesperson

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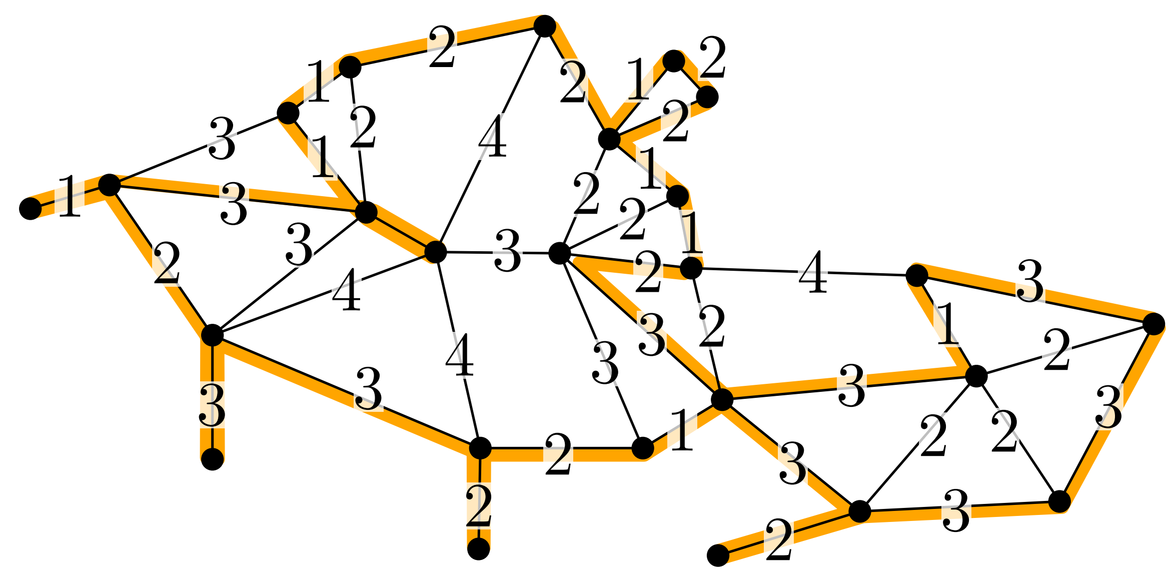
Traveling Salesperson Problem (TSP)

Input: Simple **weighted** undirected graph $G = (V, E, \omega)$, where $\omega\colon E \to \mathbb{N}$ and a **budget** $B \in \mathbb{N}$.

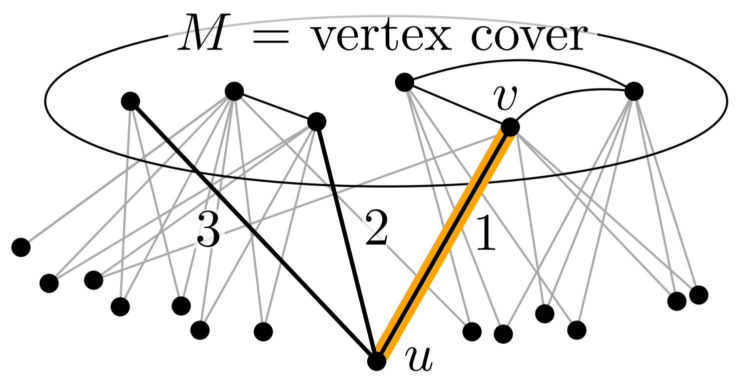
Output: Is there a **closed walk** R that visits all vertices and has the total weight at most B?

- TSP is an NP-hard problem
- it is FPT with respect to treewidth



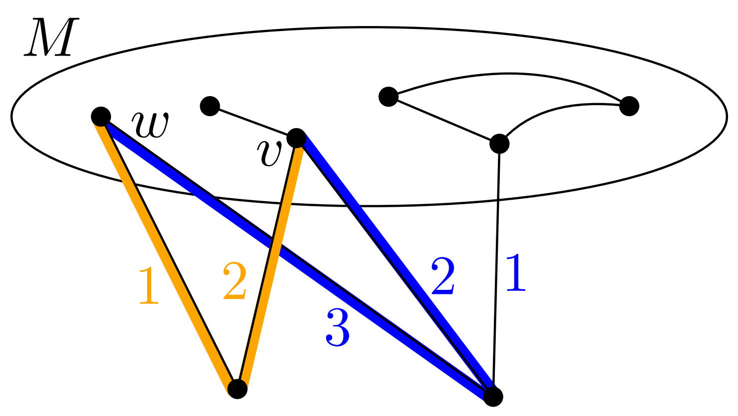
Vertex Cover Number

- vertices outside of the vertex cover M have a cheapest way to connect to M



connect u with v using a total weight 2

- connecting all vertices in the cheapest way may not give a connected solution
- "pay" an additional fee to some vertices to change their connections so that the solution is connected



pay 1 or **pay 3** to connect w with v

- retain M and a polynomial number of such vertices for each (v, w) pair
- → polynomial kernel

Our results

Vertex Cover Number
Remove k vertices to obtain an independent set.
TSP has $\mathcal{O}(k^{16})$ kernel.

Feedback Edge Set No.
Remove k edges so that no cycles are left.

Mod. to Const. Paths
Remove k vertices to obtain contant-length paths
kernel from ↓ result

Mod. to Const. Comps.
TSP has $k^{\mathcal{O}(r)}$ kernel where r is size of left connected components.

Feedback Vertex Set No.
Remove k vertices so that no cycles are left.

Fractioning Number
Remove k vertices so that components of size $\leq k$ remain.
no polynomial kernel

Mod. to Disjoint Cycles
Remove k vertices so that disjoint cycles are left.

Treewidth

arXiv

Feedback Edge Set No.

- leaves always have a clear solution
- chains of degree 2 vertices have the number of possibilities small and can be modelled with smaller subgraphs
- similar reductions also work for the generalized TSP (see box at the bottom)
- exhaustive application gives a polynomial kernel

Negative results

- no polynomial kernel for TSP with respect to the **fractioning number** unless polynomial hierarchy collapses
- no polynomial kernel with respect to the combined parameter **treewidth and maximum degree** unless polynomial hierarchy collapses
- unweighted Subset TSP with respect to the **modulator to disjoint cycles** is WK[1]-hard ⇒ no polynomial kernel

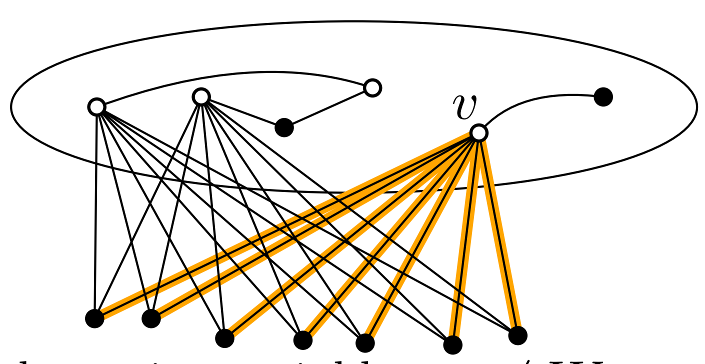
EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research, Development and Education

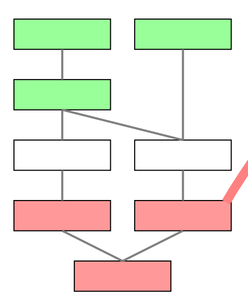
The authors acknowledge the support of the OP VVV MEYS funded project CZ.02.1.01/0.0/0.0/16.019/0000765 "Research Center for Informatics" and the Grant Agency of the Czech Technical University in Prague funded grant No. SGS20/208/OHK3/3T/18.

Generalizations

Subset TSP

- has a set of waypoints $W \subseteq V$ (full) that need to be traversed

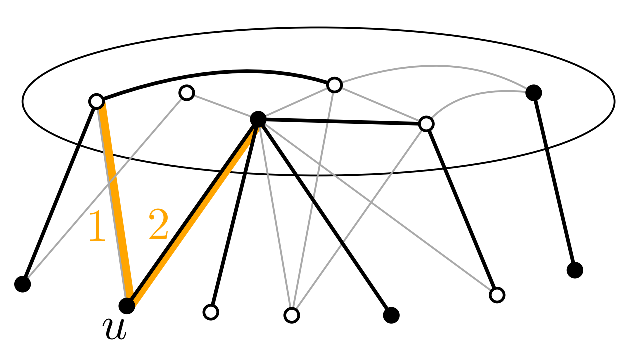


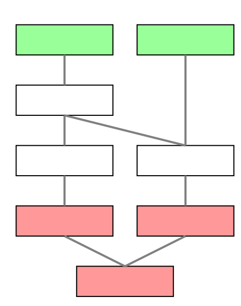


- enough vertices neighbor $v \notin W$ → reroute the solution through it
- polynomial kernel w.r.t. the modulator to constant paths

Waypoint Routing Problem

- has a capacity $c\colon E \to \mathbb{N}$ for every edge





- WRP can be reduced to capacities 1 (thin) and 2 (thick)
- polynomial kernel with respect to the vertex cover number